Topologically massive nonabelian BF models in arbitrary space-time dimensions

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Abstract

This work extends to the D-dimensional space-time the topological mass generation mechanism of the nonabelian BF model in four dimensions. In order to construct the gauge invariant nonabelian kinetic terms for a (D-2)-form B and a 1-form A, we introduce an auxiliary (D-3)-form V. Furthermore, we obtain a complete set of BRST and anti-BRST transformation rules of the fields using the so called horizontality condition, and construct a BRST/anti-BRST invariant quantum action for the model in D-dimensional space-time.

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Antisymmetric tensor gauge fields appear naturally in string theory and play an important role in dualization [1]. They are also fundamental in realization of Schwarz topological field theory through a BF term, where B is a two form gauge field and F the field strength of the one form gauge field A. The BF term can be abelian or nonabelian, can live in any dimension, and is a dimensional generalization of the Chern-Simons term. Some time ago, Allen, Bowick and Lahiri [2], using the BF term, showed that it is possible to give mass to abelian vector gauge fields without the Higgs field in four dimensions. This interesting mechanism is known as topological mass generation (TMG). This is a modified form of the well known topological mass mechanism introduced by Deser and Jackiw in the abelian gauge models with Chern-Simons term [3]. In the three-dimensional case a similar mechanism exists for generating mass to the vector field and alternatively to a scalar and two form field [4,5]. Recently, Hwang and Lee [6], showed that it is possible to construct the nonabelian TMG in four dimensions with the addition of an auxiliary vector field. This auxiliary field is necessary to eliminate a constraint that appears in the nonabelian version of TMG.

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Dimensional generalizations of BF models have been considered in refs. [7–9]. These studies considering strictly Schwarz-type topological models - which have a classical gauge fixed action written as the sum of a gauge-invariant term and a BRST-invariant term [10] - are focused on perturbative renormalization, symmetry content and formal aspects of BF models. More recently, Smailagic and Spallucci, have studied the dualization of abelian [11] and nonabelian [12] BF models of arbitrary $p-$forms to a Stueckelberg-like massive gauge invariant theories.

However our main purpose in this letter resides in a slightly different context. Here we present a D-dimensional generalization from the nonabelian topological mass generation in four dimensions. As mentionated above this mechanism was introduced by Allen, Bowick and Lahiri for the abelian BF model in the four dimensional space-time and later applied for its nonabelian version [6]. Using the BRST/anti-BRST formalism we construct here a framework which consistently prove how the auxiliary fields are required and unties the constraints in the D-dimensional case.

Our starting Lagrangian describes a nonabelian BF model in D-dimensions. Consider the field theory of a real-valued $(D-2)-$form field B and a real-valued $1-$form field A defined on a D-dimensional space-time manifold $\mathcal{M}_D$ with metric $g_{\mu\nu} = \mathrm{diag}(- + + \cdots + ++)$. Augmented with propagation terms for the gauge fields A and B, we have

$$S = \int_{\mathcal{M}_D} \mathrm{Tr}\left(\frac{1}{2} H \wedge^* H + m B \wedge F - \frac{1}{2} F \wedge^* F\right), \tag{1}$$

where $F = dA + A \wedge A$ and $H = DB = dB + [A, B]$ are the field strengths of A and B respectively, $d = dx^\mu (\partial / \partial x^\mu)$ is the exterior derivative and $*$ is the Hodge star operator. Also $A = A^a T^a$, $B = B^a T^a$, where T^a are generators of a Lie algebra $\mathcal{G}$ of a semi-simple Lie group G [1]. Note that A is a G-connection $1-$form field on $\mathcal{M}_D$. The adjoint operator acting in a $p-$form can be written as $d^\dagger = (-1)^{Dp+D} * d *$ (for a Lorentzian manifold) and the Laplacian operator reads as $\partial^2 = d\, d^\dagger + d^\dagger d$ [13].

The $B \wedge F$ term in (1) is invariant under the gauge transformations

$$\begin{aligned} \delta A &= D\theta = d\theta + [A, \theta], \\ \delta B &= D\Omega + [B, \theta], \end{aligned} \tag{2}$$

where θ is a $0-$form and Ω is a $(D-3)-$form Lie algebra valued. However as in four dimensional case, the action (1) is not invariant under (2), due to the fact that H does not transform as $\delta H = [H, \theta]$. In fact, using (2), the transformation of H is

$$\delta H = [F, \Omega] + [H, \theta]. \tag{3}$$

In order to circumvent this problem, at least classically, we introduce an auxiliary $(D-3)-$form V, and redefine H to be

$$\mathcal{H} = H + [F, V], \tag{4}$$

[1] The commutator between two Lie algebra valued forms P and Q is defined by $[P, Q] = P \wedge Q - (-1)^{d(P)d(Q)} Q \wedge P$, where $d(X)$ is the form degree of X.

with V transforming as

$$\delta V = -\Omega + [V, \theta]. \tag{5}$$

It is important to remark that for any dimension of space-time, the covariant derivative must be constructed with a $1-$form A [14].

This procedure is a generalization of the mechanism first introduced by Thierry-Mieg *et al.* [14,15] who detected the obstruction to the nonabelianization of the term $H \wedge^* H$. As pointed out by Hwang and Lee [6] in four dimensional case, the equation of motion given by the action (1), namely $D^*H + mF = 0$, gives the constraint

$$DD^*H = [F, {}^*H] = 0 \tag{6}$$

which is solved when we introduce the auxiliary $(D-3)-$form V.

To implement the quantization of the model, we have to construct the BRST and anti-BRST symmetry .

In the work of Thierry-Mieg and Ne'eman [14], a geometrical BRST quantization scheme was developed where the base space is extended to a fiber bundle space so that it contains unphysical (fiber-gauge orbit) directions and physical (space-time) directions. Using a double fiber bundle structure Quiros *et al.* [16] extended the principal fiber bundle formalism in order to include anti-BRST symmetry. Basically the procedure consists in extending the space-time to take into account a pair of scalar anticommuting coordinates denoted by y and $\overline{y}$ which correspond to coordinates in the directions of the gauge group of the principal fiber bundle. Then the so-called "horizontality condition" is imposed. This condition enforces the curvature components containing vertical (fiber) directions to vanish. Hence only the horizontal components of physical curvature in the extended space survive.

Let us define the following form fields in the extended space and valued in the Lie algebra $\mathcal{G}$ of the gauge group:

$$\widetilde{A} = A + c + \bar{c}, \tag{7}$$
$$\widetilde{F} = \tilde{d}\widetilde{A} + \widetilde{A} \wedge \tilde{A}, \tag{8}$$
$$\widetilde{B} = \textstyle\sum_{k=0}^{D-2} \sum_{n=0}^{k} B_{D-2-k}^{(n)(k-n)}, \tag{9}$$
$$\widetilde{\mathcal{H}} = \widetilde{D}\widetilde{B} + \left[\widetilde{F}, \widetilde{V}\right], \tag{10}$$
$$\widetilde{V} = \textstyle\sum_{k=0}^{D-3} \sum_{n=0}^{k} V_{D-3-k}^{(n)(k-n)}, \tag{11}$$
$$\widetilde{D} = \tilde{d} + [\widetilde{A}, \ \ ] , \tag{12}$$
$$\tilde{d} = d + s + \bar{s}. \tag{13}$$

Here we identify the components in unphysical directions with new fields, namely, c ($\overline{c}$) as ghosts (antighosts) in the case of the field A. There are $D(D-1)/2$ and $(D-1)/(D-2)/2$ components in $\widetilde{B}$ and $\widetilde{V}$ respectively. In the expansion of $\widetilde{B}$ and $\widetilde{V}$, the upper indices n and $k-n$ are respectively the ghost number and the antighost number of the $(D-2-k)-$form $B_{D-2-k}^{(n)(k-n)}$ and $(D-3-k)-$form $V_{D-3-k}^{(n)(k-n)}$, having a total degree $(D-2)$ and $(D-3)$.

Note that when we treat two odd "extended" forms, the [,] must be reading as an anticommutator.

Furthermore, we call attention for the necessary presence of the auxiliary $(D-3)-$form V field. The exterior derivatives in the gauge group directions are denoted by $s = dy^N(\partial/\partial y^N)$ and $\overline{s} = d\overline{y}^N(\partial/\partial \overline{y}^N)$.

The horizontality condition, or equivalently, the Maurer-Cartan equation for the field strengths F and $\mathcal{H}$ can be written as

$$\widetilde{\mathcal{H}} = \mathcal{H}, \tag{14}$$
$$\widetilde{F} = F. \tag{15}$$

The expansion of (15), gives us the well known BRST and anti-BRST of the gauge field A, the ghost c and the anti-ghost $\bar{c}$:

$$\begin{aligned} sA &= -Dc, \quad \bar{s}A = -D\bar{c} \\ sc &= -cc, \quad \bar{s}\bar{c} = -\bar{c}\bar{c} \\ s\bar{c} &+ \bar{s}c = -[c, \bar{c}] \end{aligned} \tag{16}$$

The transformation BRST and anti-BRST in the last equation of (16), are unknown. We must introduce an auxiliary field b, in order to fix completely those transformations

$$s\bar{c} = b, \quad \bar{s}c = -b - [c, \bar{c}], \quad sb = 0, \quad \bar{s}b = -[\bar{c}, b] \tag{17}$$

Now, expanding the equation (14) into a basis of same ghost number and form degree, we have

$$sB^{(k)(0)}_{D-2-k} = -DB^{(k+1)(0)}_{D-3-k} - [c, B^{(k)(0)}_{D-2-k}] - [F, V^{(k+1)(0)}_{D-4-k}], \tag{18}$$

$$\bar{s}B^{(0)(k)}_{D-2-k} = -DB^{(0)(k+1)}_{D-3-k} - [\bar{c}, B^{(0)(k)}_{D-2-k}] - [F, V^{(0)(k+1)}_{D-4-k}], \tag{19}$$

for $0 \leq k \leq D-2$, and

$$\begin{aligned} &DB^{(n)(k-n+1)}_{D-3-k} + [c, B^{(n-1)(k-n+1)}_{D-2-k}] + [\bar{c}, B^{(n)(k-n)}_{D-2-k}] + \\ &sB^{(n-1)(k-n+1)}_{D-2-k} + \bar{s}B^{(n)(k-n)}_{D-2-k} + [F, V^{(n)(k-n+1)}_{D-4-k}] = 0, \end{aligned} \tag{20}$$

for $1 \leq n \leq k$, $1 \leq k \leq D-2$.

We have $(D-1)(D-2)$ not defined s and $\bar{s}$ transformations in (20). Therefore we must introduce a set of $(D-1)(D-2)/2$ auxiliary fields, namely $\omega's$, in order to fix the transformation rule completely.

$$sB^{(n-1)(k-n+1)}_{D-2-k} = \omega^{(n)(k-n+1)}_{D-2-k} \tag{21}$$

$$\begin{aligned} \bar{s}B^{(n)(k-n)}_{D-2-k} &= -DB^{(n)(k-n+1)}_{D-3-k} - \omega^{(n)(k-n+1)}_{D-2-k} - \\ &[c, B^{(n-1)(k-n+1)}_{D-2-k}] - [\bar{c}, B^{(n)(k-n)}_{D-2-k}] - [F, V^{(n)(k-n+1)}_{D-4-k}] \end{aligned} \tag{22}$$

The condition (14) leads us to

$$\widetilde{B} + \widetilde{D}\widetilde{V} = B + DV, \tag{23}$$

which now yields the BRST/anti-BRST transformation rule for the components of $\widetilde{V}$

$$sV^{(k)(0)}_{D-3-k} = -DV^{(k+1)(0)}_{D-4-k} - [c, V^{(k)(0)}_{D-3-k}] - B^{(k+1)(0)}_{D-3-k}\,, \tag{24}$$

$$\bar{s}V^{(0)(k)}_{D-3-k} = -DV^{(0)(k+1)}_{D-4-k} - [\bar{c}, V^{(0)(k)}_{D-3-k}] - B^{(0)(k+1)}_{D-3-k}\,, \tag{25}$$

for $0 \leq k \leq D-3$, and

$$\begin{gathered} DV^{(n)(k-n+1)}_{D-4-k} + sV^{(n-1)(k-n+1)}_{D-3-k} + \bar{s}V^{(n)(k-n)}_{D-3-k} + \\ [c, V^{(n-1)(k-n+1)}_{D-3-k}] + [\bar{c}, V^{(n)(k-n)}_{D-3-k}] + B^{(n)(k-n+1)}_{D-3-k} = 0, \end{gathered} \tag{26}$$

for $1 \leq n \leq k$ and $1 \leq k \leq D-3$. Again, these equations do not fix the BRST/anti-BRST transformation rule, so we need a set of $(D-2)(D-3)/2$ auxiliary fields η:

$$sV^{(n-1)(k-n+1)}_{D-3-k} = \eta^{(n)(k-n+1)}_{D-3-k} \tag{27}$$

$$\begin{aligned} \bar{s}V^{(n)(k-n)}_{D-3-k} \quad &= -\eta^{(n)(k-n+1)}_{D-3-k} - DV^{(n)(k-n+1)}_{D-4-k} - \\ &[c, V^{(n-1)(k-n+1)}] - [\bar{c}, V^{(n)(k-n)}_{D-3-k}] - B^{(n)(k-n+1)}_{D-3-k}. \end{aligned} \tag{28}$$

In order to obtain the BRST/anti-BRST transformations of the auxiliary fields ω and η, we use the nilpotency condition of s and $\bar{s}$:

$$s\omega^{(n)(k-n+1)}_{D-2-k} = 0 \tag{29}$$

$$\begin{aligned} \bar{s}\omega^{(n)(k-n+1)}_{D-2-k} \quad &= -[Dc, B^{(n-1)(k-n-2)}_{D-3-k}] - D\omega^{(n)(k-n+2)}_{D-3-k} - \\ &[B^{(n-1)(k-n+1)}_{D-2-k}, b] - [\bar{c}, \omega^{(n)(k-n+1)}_{D-2-k}] - [c, \omega^{(n-1)(k-n+2)}_{D-2-k}] - \\ &[cc, B^{(n-2)(k-n+2)}_{D-2-k}] + [F, \eta^{(n)(k-n+2)}_{D-4-k}] \end{aligned} \tag{30}$$

$$s\eta^{(n)(k-n+1)}_{D-3-k} = 0 \tag{31}$$

$$\begin{aligned} \bar{s}\eta^{(n)(k-n+1)}_{D-3-k} \quad &= -[Dc, V^{(n-1)(k-n+2)}_{D-4-k}] - D\eta^{(n)(k-n+1)}_{D-3-k} + \\ &\omega^{(n)(k-n+2)}_{D-3-k} - [c, \eta^{(n-1)(k-n+2)}_{D-3-k}] - [\bar{c}, \eta^{(n)(k-n+1)}_{D-3-k}] - \\ &[cc, V^{(n-2)(k-n+2)}_{D-3-k}] - [V^{(n-1)(k-n+1)}_{D-3-k}, b] \end{aligned} \tag{32}$$

Therefore, a complete set of BRST and anti-BRST equations, namely, eqs. (16-19), (21,22), (24-25), and (27-32), associated with the classical symmetry (2), were obtained.

Finally, the topologically massive nonabelian BF model in $D-$dimensions BRST/anti-BRST invariant can be written as

$$S_{cl} = \int_{\mathcal{M}_D} \mathrm{Tr}\left(\frac{1}{2}\mathcal{H}\wedge{}^{*}\mathcal{H} + mB\wedge F - \frac{1}{2}F\wedge{}^{*}F\right). \tag{33}$$

In the expression above we could be use $\mathcal{B} = B + DV$. However, in the action (33), $mB \wedge F$ differs from $m\mathcal{B} \wedge F$ by a total derivative term as occurs in the four dimensional case.

The simplest scenario to study mass generation is to consider the equations of motion of the action (33). Namely

$$(-1)^{(D-1)} D^{*}\mathcal{H} + mF = 0 \tag{34}$$

and

$$D^{*}F = mDB + [B,^{*}\mathcal{H}] + D[V,^{*}\mathcal{H}]. \tag{35}$$

$$[F,^{*}\mathcal{H}] = 0 \tag{36}$$

The last equation corresponds to the constraint (6), but now appears as the equation of motion for the auxiliary field V.

Considering only linear terms for the fields, from equations (34) and (35) we get:

$$\left(\partial^2 - m^2\right) H = 0, \tag{37}$$

$$\left(\partial^2 - m^2\right) F = 0. \tag{38}$$

which exhibit mass generation for H and F. Note that, in our metric, $\partial^2 = -\partial_t^2 + \nabla^2 = -\Box$.

We now propose as a quantized action of our $D-$dimensional nonabelian BF model the following expression

$$S = S_{cl} + \int \mathrm{Tr} s\overline{s} \left(A \wedge^{*} A + \alpha c \wedge^{*} \overline{c} + \sum_{k=0}^{D-2} \sum_{n=0}^{k-1} \lambda_{kn} B_{D-2-k}^{(n)(k-n)} \wedge^{*} B_{D-2-k}^{(k-n)(n)} \right), \tag{39}$$

where α and λ_{kn} are gauge parameters.

This geometrical quantization method was formulated by Thierry-Mieg and Baulieu [17] for Yang-Mills theories and later for nonabelian antisymmetric tensor gauge theories [15]. This method is particularly relevant for treating models with ghosts for ghosts and differential constraints, where the Fadeev-Popov construction does not work.

It is worth mentioning that years ago, a nonabelian dimensional generalization of topologically massive gauge theories involving antisymmetric tensor fields was proposed [18]. However it *does not describe* a BF $D-$dimensional model. In fact, that work consider $n-$form and $(D-n-1)$ form fields (which are not gauge connections) and does not present an Yang-Mills term TrF^2 . As a matter of fact, they consider a flat connection 1-form A, consequently $F = 0$. Therefore the constraints above discussed are absent from that model.

In summary, we setup a geometrical construction of the BRST/anti-BRST equations in the massive gauge-invariant nonabelian BF model in $D-$ dimensions using the horizontality condition. This procedure generalizes the method of Hwang and Lee in four dimensional space-time [6], and consistently extend to $D-$ dimensions the introduction of auxiliary fields in order to provide a nonabelianization of the propagation term of a $(D-2)-$form field.

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